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Energy Distribution in Atmospheric Muons

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Abstract: This study models the parameters of atmospheric muons that decay at altitudes ranging from 200 to 4200 meters. Using experimental data, this research determined the correlation between the most likely number of decayed muons, the altitude of decay and the distance traversed before decaying. Next, using the data obtained, this study determined the fraction of total decayed muons that exhibited a certain velocity and determined their altitude of decay. The velocities of the fractions of decayed muons were further used to determine their corresponding kinetic energies. The study also presented equations that give the relation between the kinetic energy of a muon and its altitude of decay. Furthermore, an equation relating the energy of a muon to the fraction of muons corresponding to that energy was also created in the present study. To diminish the standard deviation in the data obtained thus far, this paper also determines and plots the relation between the specific energy of atmospheric muons and the fraction of muons with a kinetic energy greater than or equal to the specified energy. Overall, this study describes, plots, and generates equations for the kinetic energies of muons and calculates the muon fraction for different energies to determine the energy distribution for atmospheric muons that decay between 200 meters and 4200 meters.

Keywords: Particle Physics; Experimental Modelling; Atmospheric Muons; Atmospheric Particle Interactions; Muon Energetics.

INTRODUCTION

Muons are second-generation leptonic elementary particles with characteristics comparable to those of electrons. While they have the same charge (-1) and spin (1/2) as an electron, they are considerably more massive—approximately 207 times greater than an electron. Muons, like leptons, lack a substructure. They have a half-life of 1.523 microseconds. Muons are most typically discovered as a result of cosmic rays, which are high-energy particles (mainly protons and alpha particles) that interact with the Earth's upper atmosphere at altitudes ranging from 10 to 15 km. We can observe these muons at sea level due to this process imparting relativistic velocity to each muon as a direct result of special relativity.

This study aimed to determine the kinetic energy distribution of atmospheric muons that are generated when cosmic rays, constituting protons, strike the upper atmosphere. The impetus for this research arises from the critical role that muons play in unravelling cosmic mysteries, particularly in discerning the mechanisms of particle acceleration and the propagation of cosmic rays throughout interstellar space. By scrutinizing the energy spectrum of atmospheric muons at varying altitudes, this study endeavours to contribute to the broader landscape of particle astrophysics and foster a deeper understanding of the cosmic processes shaping our universe.

This study aims to expand particle astrophysics by investigating the amusing energy distribution of atmospheric muons. This research tries to expand our understanding of cosmic processes and their implications for our cosmic surrounds by referencing studies that used cutting-edge experimental methodologies, advanced theoretical models, and computing skills to generate conclusions.

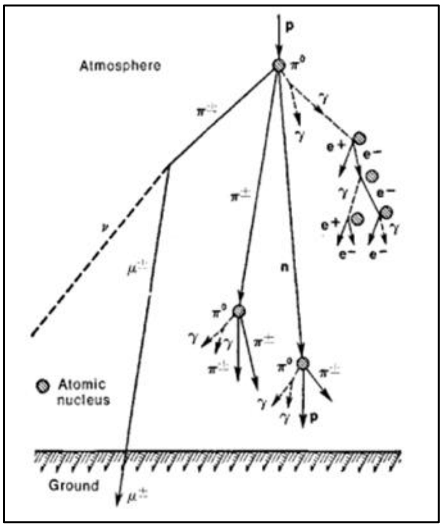

Figure 1: ***Diagram of cosmic ray showers, showing the various decays that occur upon the interaction of a cosmic ray with the atmosphere.***

Literature Review

This literature review aims to present a compendium of experimental research that has investigated the nature of relativistic processes on muons, providing insight into the substantial consequences these interactions have for our knowledge of fundamental physics. Additional research has investigated alterations in muon detection with respect to solid angle, altitude, and angle of incidence.

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This article presents one of the earliest experimental validations of time dilation, a central tenet of Einstein's theory of special relativity, involved muons. Experiments carried out in the mid-twentieth century, such as the seminal work of Rossi and Hall, indicated that the observed muon lifetimes were significantly longer than the laboratory-measured lifetimes while at rest. This disparity was a direct outcome of time delay, demonstrating that the relativistic speeds of muons caused time delay, essentially lengthening their lifespan.

The Tokyo-Edinburgh collaboration employed a detector located at high altitudes to observe cosmic-ray-induced muons. The measured muon fluxes at various altitudes attested to the relativistic effects on momentum, with higher energy muons having greater penetration depths due to time dilation and momentum conservation.

The Fermilab Muon g-2 experiment marks a milestone in the precise measurements of muon characteristics, with a special focus on the anomalous magnetic dipole moment. The experiment's determination of the precession frequency of the muon in a magnetic field offered more nuanced knowledge of the relativistic processes that regulate muon behaviour. This work not only confirmed special relativity predictions but also shed light on the velocity-dependent decay properties of muons.

The MINOS experiment at Fermilab investigated the occurrence of Lorentzian contraction in muon decay channels through the investigation of neutrino oscillations. MINOS provided empirical evidence of the impact of Lorentzian contraction on spatiotemporal aspects of muon decay by examining the decay products of muon neutrinos across long baselines. This study emphasizes the interrelated nature of relativistic effects and their impact on basic muon characteristics.

The NA48/2 group at CERN delves into the discrete link between particle energies and relativistic kinematics by studying the energy dependence of muon decay spectra. The study concentrated on charged kaon and muon decay modes, yielding important insights into the energy-dependent relativistic implications on decay probability.

A different investigation at the mu Lab used plastic scintillation detectors to calculate muon flux as a function of altitude, angle, and solid angle. The acquired data were compared to literature models, which revealed significant convergence, even with less technically rigorous models. This study highlights the possibility of improving more robust modelling methodologies by evaluating muon energy distributions at different altitudes. The experiment used four "Cosmic Watch" muon scintillation detectors, each of which had a silicon photomultiplier beneath a scintillation block. At many locations, including Mount Rose, Truckee, Mount Diablo, and Clark Kerr UC Berkeley, altitude fluctuations were investigated. Furthermore, a finely calibrated mechanical mechanism allowed for exact positioning of detectors at certain angles (0°, 30°, and 60°), adding to a thorough knowledge of muon detection dependencies.

METHODOLOGY

Different muons have different energies; hence, they move with varying velocities that correspond to their individual velocities. Due to their different velocities, different muons experience different magnitudes of time dilation and length contraction. Muons with more energy experience a greater contraction length than do those with less energy. This causes muons with more energy to decay after traversing longer distances than muons with less energy.

Based on the findings of the study conducted by A. McNicholas [1], this study will determine and plot the fraction of muons along with their kinetic energies. The muons studied range from those that reach an altitude of 200 meters to those that reach an altitude of 4200 meters. The data from the above study are presented in Table 1.

Table 1: Results of Different muons

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Serial Number (i)	Most probable value of muons detected per second (n_i)	Altitude (a_i) (meters)
1	6.3	4200
2	5.8	3950
3	5.0	3400
4	4.4	3000
5	3.8	2600
6	3.5	2200
7	2.9	1850
8	2.7	1400
9	2.5	950
10	2.2	600
11	2.0	200

According to the 2018 study titled Cosmic Ray Muon Detection by E. Boulicaut, the average altitude of atmospheric muon formation can reach 15000 meters.

Table 1 shows that the most likely number of decayed muons between 200 and 4200 meters is 4.3. To determine the most likely number of muons that decayed (d_i) between two altitudes (a_i and a_{i+1} in this case), the corresponding numbers of detected muons at both altitudes were subtracted. The altitude at which the found number of muons (d_i) decayed can be determined by taking the average of both the altitudes, a_i and a_{i+1}.

The relationship between the altitude and the number of decayed muons is given below:

$$Most\ probable\ number\ of\ muons\ decayed\ at\ altitude\ \frac{a_i + a_{i+1}}{2}\ is\ n_i - n_{i+1}.$$

where i is the serial number given in Table 1. Since muons originate at an altitude of 15000 meters and decay at an altitude of a_d meters, the distance traversed by a muon before decaying (d_{mps}) is given by equation 1.

$$d_{mps} = 15000 - a_d \quad Equation - 1$$

From the above relation, Table 2 can be constructed as follows:

Table 2: The relation between a muon's velocity and time dilation

Serial Number (i)	Most probable number of muons decayed (n_{mps})	Altitude of decay (a_d) (meters)	Distance traversed before decaying (d_{mps}) (meters)
1	0.5	4075	10925
2	0.8	3675	11325
3	0.6	3200	11800
4	0.6	2800	12200
5	0.3	2400	12600
6	0.6	2025	12975
7	0.2	1625	13375
8	0.2	1175	13825
9	0.3	775	14225
10	0.2	400	14600

The relation between a muon's velocity and time dilation is given by equation 2.

$$\triangle t = \frac{\triangle t'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad Equation - 2$$

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where $\triangle t'$ is the time elapsed from the muon's frame of reference, $\triangle t$ is the time elapsed with reference to the observer, and v is the velocity of the muon. If the muon is observed at an altitude (a), then the value of $\triangle t$ for that muon is given by equation 3.

$$\triangle t' = \frac{(15000 - d_{mps})}{v} \quad Equation - 3$$

Since muons have an average lifetime of 2.197 µs, the value of $\triangle t'$must be equal to 2.197 µs. This can be said on the basis of the assumption that a muon decays as soon as it is detected, and a similar behaviour of muons can be stated from the data in Table 2 due to quantized data measurements. By combining equations 2 and 3 and replacing $\triangle t$ inequation 2 with the RHS of equation 3, equation 4 is formed. It provides the relation between the distance traversed by a muon before decaying and the velocity of the muon.

$$\frac{(15000 - d_{mps})}{v} = \frac{\triangle t'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad Equation - 4$$

Furthermore, inputting the determined values and constants in equation 4 yields equation 5.

$$\frac{(15000 - d_{mps})}{v} = \frac{2.197 * 10^{-6}}{\sqrt{1 - \frac{v^2}{8.9575 * 10^{16}}}} \quad Equation - 5$$

By substituting the values of d_{mps} from Table 2 in equation 5, we obtain the corresponding values of the velocity of the muon. Table 3 displays the velocities of the muons corresponding to their altitude of decay, number of muons decayed, and distance traversed before decaying.

Table 3: The fraction of muons (µ_f) with a certain velocity

Serial Number (i)	Most probable number of muons decayed (n_{mps})	Altitude of decay (a_d) (meters)	Distance traversed before decaying (d_{mps}) (meters)	Velocity of muon (v) (meters per second)
1	0.5	4075	10925	$1.55546 * 10^8$
2	0.8	3675	11325	$2.28217 * 10^8$
3	0.6	3200	11800	$2.61176 * 10^8$
4	0.6	2800	12200	$2.77438 * 10^8$
5	0.3	2400	12600	$2.84660 * 10^8$
6	0.6	2025	12975	$2.88653 * 10^8$
7	0.2	1625	13375	$2.91365 * 10^8$
8	0.2	1175	13825	$2.93166 * 10^8$
9	0.3	775	14225	$2.94612 * 10^8$
10	0.2	400	14600	$2.95469 * 10^8$

The fraction of muons (μ_f) with a certain velocity can be calculated by equation 6 as follows:

$$\mu_f = \frac{n_{mps}}{4.3} \quad Equation - 6$$

where n_{mps} represents the number of decayed muons at the corresponding velocity (same serial number).
Let m_μ be the mass of the muon, which is equal to $\frac{1.507MeV}{c^2}$. The kinetic energy (E_K), in eV, of a muon is given by equation 7:

$$E_K = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) * 1.507 * 10^6 \quad Equation - 7$$

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RESULTS

Utilizing the data of existing studies on changes in the number of atmospheric muons with variations in altitude ranging from 200 meters to 4200 meters, this study determines the various parameters of atmospheric muons and their dependencies and distributions. Further, comparing the fraction of muons with the parameters eliminates the altitude of decay, location of detection, and meteorological conditions as parameters influencing the minimum and total muon energy per muon fraction.

Within the altitude range of the study area, the correlation between the most likely number of muons that decayed and the altitude of decay, the velocity relative to the Earth's surface, and the distance traversed before decaying are given in Table 3.

In summary, this study successfully modelled the correlation between atmospheric muons and their kinetic energies, altitude of decay, fraction of muons decayed and fraction of muons with a corresponding threshold energy and generated graphs of the distributions of these parameters. The data generated by this study's models of the variation in the kinetic energy of muons with altitude, fraction of muons with a specific energy, and fraction of muons with a minimum energy match the experimental data with errors of 1%, 9.4% and 0.021%, respectively.

CONCLUSION

In conclusion, this study provides a thorough examination of atmospheric muon decay at altitudes ranging from 200 to 4200 metres. This research effectively established connections between the most likely amount of decaying muons, the height of decay, and the distance travelled prior to decay using experimental data. The developed equations for the link between kinetic energy and decay altitude provide insights into the energy distribution of atmospheric muons.

Furthermore, this investigation delves into the fractionation of decayed muons based on their velocities, subsequently determining their altitude of decay and corresponding kinetic energies. The equations presented herein offer a mathematical framework for understanding the interplay between muon energy and atmospheric altitude.

To enhance the precision and diminish the standard deviations in the dataset, this study introduces a relation between specific energy thresholds and the corresponding fractions of muons possessing kinetic energies equal to or greater than the specified threshold. This refinement not only bolsters the accuracy of the results but also contributes to a more nuanced understanding of the energy distribution of atmospheric muons within the defined altitude range.

Overall, this research aims to contribute to the evolving landscape of muon physics, providing a comprehensive framework for characterizing and modelling the energy distribution of atmospheric muons. The presented equations and correlations are tools for future studies in particle astrophysics and further our understanding of the interactions between muons and the atmosphere. As technology and methodologies continue to advance, this study lays a foundation for more refined investigations into the relativistic behaviours of muons and their energy distribution.

REFERENCES

1. McNichols, A. T. *Variable Altitude Muon Detection and Energy Dependence of Cosmic Ray Muons*. 2014.
2. Deck, E. E., et al. *Angular and Altitude Dependence of Cosmic Ray Muons*.
3. Easwar, N., and D. A. Macintire. "Study of the Effect of Relativistic Time Dilation on Cosmic Ray Muon Flux—An Undergraduate Modern Physics Experiment." *American Journal of Physics*, vol. 59, 1991, pp. 589–592.
4. Frisch, D. H., and J. H. Smith. "Measurement of the Relativistic Time Dilation Using μ-Mesons." *American Journal of Physics*, vol. 31, 1963, pp. 342–355.
5. Premovic, Pavle. "Relativistic Time Dilation and the Muon Experiment." *The General Science Journal*, 2015.
6. Supanitsky, A. D. "Estimation of the Number of Muons with Muon Counters." *Astroparticle Physics*, vol. 127, 2021, p. 102535.
7. Gesualdi, F., and A. D. Supanitsky. "Estimation of the Number of Counts on a Particle Counter Detector with Full Temporal Resolution." *The European Physical Journal C*, vol. 82, 2022.

Publisher: SARC Publisher

8. Boulicaut, E. L. *Cosmic Ray Muon Detection*, 2018.
9. Kox, A. J. *On the Influence of Gravitation on the Propagation of Light*, 2011.
10. Tangherlini, F. R. *Muon Mass*. Coll. of the Holy Cross, Worcester, MS, 1973.

Publisher: SARC Publisher