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Faraday Complex Matrix in Terms of Dirac Matrices

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Abstract: For the Faraday complex matrix, we exhibit its expansion in terms of the Dirac matrices.
Keywords: Gamma matrices, Faraday complex matrix, Pauli matrices, Electromagnetic field.

INTRODUCTION

In [López-Bonilla, J. *et al*., 2021], for an arbitrary matrix M_{4x4}, it was obtained its expression in terms of the sixteen Dirac matrices, in the standard representation [Good, R. H. 1955; Bagrov, V.G. *et al*., 2014]:

$$I, \qquad \gamma^{\mu}, \qquad \gamma^{5}, \qquad \gamma^{\mu}\gamma^{5}, \qquad \sigma^{\mu\nu}, \tag{1}$$

in fact:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = a_0\,\gamma^0\gamma^5 + b_0\,I + c_0\,\gamma^0 + id_0\,\gamma^5 + b_1\,\sigma^{23} + b_2\,\sigma^{31} + b_3\,\sigma^{12} +$$

$$+ \sum_{j=1}^{3} (a_j\,\gamma^j + c_j\,\gamma^j\gamma^5 + d_j\,\sigma^{oj}), \tag{2}$$

$$= \sum_{k=0}^{3} [\, a_k \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} + b_k \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} + c_k \begin{pmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{pmatrix} + i\, d_k \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}],$$

in terms of the Pauli matrices [Leite-Lopes, J. 1977] with $\sigma_0 = I_{2x2}$, therefore:

$$A = \sum_{k=0}^{3} (b_k + c_k)\,\sigma_k, \qquad B = \sum_{k=0}^{3} (a_k + i\,d_k)\,\sigma_k,$$

$$C = \sum_{k=0}^{3} (-a_k + i\,d_k)\,\sigma_k, \qquad D = \sum_{k=0}^{3} (b_k - c_k)\,\sigma_k,$$

whose solution is given by:

$$a_k = \frac{1}{4}\, tr\,[(B - C)\,\sigma_k], \qquad d_k = -\frac{i}{4}\, tr\,[(B + C)\,\sigma_k],$$
$$k = 0, 1, 2, 3, \tag{3}$$
$$c_k = \frac{1}{4}\, tr\,[(A - D)\,\sigma_k], \qquad b_k = \frac{1}{4}\, tr\,[(A + D)\,\sigma_k].$$

That is, we have M, hence we know A, B, C, D, then we calculate $(A \pm D)$ and $(B \pm C)$ and their products with the Pauli matrices, and finally we obtain the corresponding traces to determine the coefficients in the expansion (2) in according with (3).

We consider that the results (3) for the expansion (2) can be useful to study the matrix that transforms a Dirac spinor under Lorentz mappings [López-Bonilla, J. *et al*., 2020; López-Bonilla, J. *et al*., 2021], to analyze the Frenet–Serret curvature matrix for the motion of point particles in Minkowski spacetime [Gürsey, F. 1957; López-Bonilla, J. *et al*., 1997], and also to represent an arbitrary Lorentz transformation [López-Bonilla, J. *et al*., 2020; López-Bonilla, J. *et al*., 2021].

Faraday complex matrix

Minkowski [1908] introduced the skew-symmetric matrix:

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$$F = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -cB_3 & cB_2 \\ E_2 & cB_3 & 0 & -cB_1 \\ E_3 & -cB_2 & cB_1 & 0 \end{pmatrix}, \qquad (4)$$

where $\vec{E} = (E_1, E_2, E_3)$ and $\vec{B} = (B_1, B_2, B_3)$ are the electric and magnetic fields expressed in the MKS system of units, respectively; the corresponding dual matrix is obtained directly from (4) making the changes:

$$\vec{E} \to -c\vec{B}, \qquad c\vec{B} \to \vec{E}, \qquad (5)$$

which is a symmetry of the vacuum Maxwell equations:

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}, \qquad (6)$$

therefore:

$$^*F = \begin{pmatrix} 0 & cB_1 & cB_2 & cB_3 \\ -cB_1 & 0 & -E_3 & E_2 \\ -cB_2 & E_3 & 0 & -E_1 \\ -cB_3 & -E_2 & E_1 & 0 \end{pmatrix}. \qquad (7)$$

Then the Faraday complex matrix is given by:

$$S = F + i\, {}^*F = \begin{pmatrix} 0 & iF_1 & iF_2 & iF_3 \\ -iF_1 & 0 & -F_3 & F_2 \\ -iF_2 & F_3 & 0 & -F_1 \\ -iF_3 & -F_2 & F_1 & 0 \end{pmatrix}, \qquad (8)$$

involving the components of the Riemann [Riemann, B. 1901]-Silberstein [Silberstein, L. 1907; Silberstein, L. 1907] complex vector (thus named by Bialynicki-Birula [Bialynicki-Birula, I. 1996]) [Hamdan, N. *et al*., 2008]:

$$\vec{F} = c\,\vec{B} + i\,\vec{E}; \qquad (9)$$

and the comparison of (8) with (2) gives the matrices:

$$A + D = \begin{pmatrix} 0 & (i-1)F_1 \\ -(i-1)F_1 & 0 \end{pmatrix}, \qquad A - D = \begin{pmatrix} 0 & (i+1)F_1 \\ -(i+1)F_1 & 0 \end{pmatrix},$$
$$(10)$$
$$B + C = \begin{pmatrix} 0 & (i+1)F_3 \\ -(i+1)F_3 & 0 \end{pmatrix}, \qquad B - C = \begin{pmatrix} 2iF_2 & (i-1)F_3 \\ (i-1)F_3 & 2F_2 \end{pmatrix},$$

which can be multiplied by the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (11)$$

and thus (3) implies the following non-zero values:

$$a_0 = p\,F_2, \quad a_1 = -\bar{p}\,F_3, \quad a_3 = -\bar{p}\,F_2, \quad b_2 = -p\,F_1, \quad c_2 = -\bar{p}\,F_1, \quad d_2 = p\,F_3, \qquad (12)$$

where $p = \frac{1}{2}(1+i)$ and $\bar{p} = \frac{1}{2}(1-i)$. Hence with (2) and (12) we obtain the expansion of the Faraday complex matrix (8) in terms of Dirac matrices:

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$$S = p\,(-F_1\,\sigma^{31} + F_2\,\gamma^0\gamma^5 + F_3\,\sigma^{02}) - \bar{p}\,(F_1\,\gamma^2\gamma^5 + F_2\,\gamma^3 + F_3\,\gamma^1). \qquad (13)$$

If we apply the duality operation (5) to (9) and (13) we deduce the changes:

$$F_j \;\rightarrow\; -i\,F_j\,, \qquad S \;\rightarrow\; -i\,S\,, \qquad (14)$$

which are important to study the electromagnetic field via the spinor and Newman-Penrose formalisms [O'Donnell, P. 2003; Hernández-Galeana, A. *et al*., 2015].

CONCLUSIONS

The results (2) and (3) are interesting by their application to quantum mechanics, relativity, differential geometry and electrodynamics, among others topics. It can be useful to obtain expressions for the coefficients of the expansion (2) in the Chiral and Majorana representations [López-Bonilla, J. *et al*., 2021].

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