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On the Gandhi's recurrence relation for colour partitions

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Abstract: We show that the Gandhi's recurrence relation for colour partitions implies the Osler-Hassen-Chandrupatla's expression for the divisor function.

Keywords: Partition function, Colour partitions, Sum of divisors function.

INTRODUCTION

Gandhi (Gandhi, J. M. 1963; Lazarev, O. 2010) showed the following recurrence relation for the colour partitions:

$$p_r(n) = -\frac{r}{n}\sum_{k=0}^{n} p_r(n-k)\,\sigma(k), \qquad n \geq 1, \tag{1}$$

Involving the divisor function $\sigma(m)$. in the next Section we exhibit that (1) implies the Osler-Hassen-Chandrupatla's identity (Osler, T. J. *et al*., 2007; López-Bonilla, J. *et al*., 2021).

Expression of Osler *et al.*,

For $r = -1$ the relation (1) implies the known property (Osler, T. J. *et al*., 2007; López-Bonilla, J. *et al*., 2021; Stanley, R. P. 1999; Ballantine, C. and M. Merca. 2017).

$$n\,p(n) = \sum_{k=0}^{n} p(n-k)\,\sigma(k) = \sum_{j=0}^{n} p(j)\,\sigma(n-j), \tag{2}$$

Where $p(n) = p_{-1}(n)$ is the partition function.

If $r = 1$, the Gandhi's expression (1) gives the Osler-Hassen-Chandrupatla's recurrence (Osler, T. J. *et al*., 2007).

$$\sigma(n) = -n\,a_n - \sum_{k=1}^{n-1} a_{n-k}\,\sigma(k), \qquad n \geq 2, \tag{3}$$

Where it was applied the result (López-Bonilla, J. *et al*., 2021; R., J. *et al*., 2021).

$$p_1(n) = a_n = \begin{cases} 0 & if \quad n \neq \frac{N}{2}(3N+1), \\ (-1)^N & if \quad n = \frac{N}{2}(3N+1), \end{cases} \qquad N = 0, \pm 1, \pm 2, \ldots \tag{4}$$

Jha, S. (2020) obtained the interesting identity:

$$\sigma(n) = n\sum_{r=1}^{n} \frac{(-1)^r}{r}\binom{n}{r} p_r(n), \qquad n \geq 1, \tag{5}$$

Which can be considered as the inversion of (1).

Finally, with (1) and (3) it is possible to deduce the following property:

$$\frac{r}{r+1}\,p_{r+1}(n) = \frac{1}{n}\sum_{k=0}^{n} k\,a_{n-k}\,p_r(k), \qquad r \geq 0,\; n \geq 1, \tag{6}$$

Similar to the relation (López-Bonilla, J. *et al*., 2021).

$$p_{r+1}(n) = \sum_{k=0}^{n} a_{n-k}\;p_r(k). \tag{7}$$

CONCLUSIONS

We showed that the Gandhi's formula involving the colour partitions and the divisor function implies important identities related with the partition function.

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