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A note on the ratio of consecutive prime numbers

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Abstract: We accept that the Andrica's conjecture (1986) is correct and it is applied to a result of Sándor (2002) involving consecutive prime numbers, and thus we obtain a bound for the ratio of such numbers.
Keywords: Primes numbers, Bertrand's theorem, Andrica's conjecture.

INTRODUCTION

We know the famous conjecture proposed by Andrica (Andrica, D. 1986) for consecutive prime numbers:

$$\sqrt{p_{n+1}} - \sqrt{p_n} < 1 , \qquad (1)$$

And the result deduced by Sándor (Sándor, J. 2002).

$$\sqrt{p_{n+1}} - \log p_{n+1} > \sqrt{p_n} - \log p_n , \qquad n \geq 3, \qquad (2)$$

Then in the following Section we employ (1) and (2) to obtain a bound for the ratio of consecutive primes.

An implication from the expressions of Sándor and Andrica

The application of (1) in (2) gives the relation $0 < \log p_{n+1} - \log p_n < 1$, that is:

$$\frac{p_{n+1}}{p_n} < e = 2.718\ 281\ 828 \ldots \qquad (3)$$

Which is compatible with the Bertrand's theorem (Dickson, L. E. 2005).

As an alternative to (1), Visser (Visser, M. 2018) showed the inequality:

$$\sqrt{p_{n+1}} - \sqrt{p_n} < \frac{11}{25} \log p_n , \qquad (4)$$

Then (2) implies the following ratio for consecutive prime numbers:

$$\frac{p_{n+1}}{p_n} < (p_n)^{0.44} , \qquad (5)$$

Which is less precise than (3).

CONCLUSIONS

It is simple to obtain (3) if we accept that the Andrica's conjecture is correct, which gives a bound for the ratio of consecutive primes in agreement with the Bertrand's theorem.

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