Sarcouncil Journal of Applied Sciences
ISSN(Online): 2945-3437
Volume- 01| Issue- 01| 2021

Short Communication

Received: 08-12-2021 | **Accepted:** 21-12-2021 | **Published:** 29-12-2021

Counting number of subgroups of order p of group G= Zp⊕ Zp⊕ …⊕ Zp

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Abstract: This article belongs to subject Abstract Algebra. This article contains counting of some possible orders number of subgroup of group G, where G is external direct product of groups G= Zp⊕ Zp⊕ …⊕ Zp (n times), where p is prime number. As p is prime Z_p is cyclic group of order p.

Keywords: External direct product, cyclic group, Sylow theorem.

INTRODUCTION

Let, G= Zp⊕ Zp⊕ …⊕ Zp be a group of order p^n.

Now, possible orders of subgroups of group G are, 1, p, p^2, p^3, … , p^{n-1}, p^n.

Group G has exactly one subgroup of order 1 and exactly one subgroup of order p^n (trivial Subgroups).

Proposition: Number of subgroups of order p (p is prime) of G= Zp Zp … Zp are $p^{n-1} + p^{n-2} + p^{n-3} + \cdots + p + 1$.

Proof: As p is prime number. Therefore subgroup of order p is cyclic subgroup. G= Zp⊕ Zp⊕ …⊕ Zp has elements of order 1 and p only.

Total number of elements in G = p^n

Number of elements of order 1 in G = 1 (identity only) Number of elements of order p in G = $p^n - 1$

And, number of cyclic subgroup of order d of group G= $\frac{\text{number of elements of order d in G}}{\phi(d)}$

∴ Number of cyclic subgroup of order p of G= $\frac{\text{number of elements of order p in G}}{\phi(p)}$

$$\therefore \text{number of cyclic subgroup of order p of G} = \frac{p^n - 1}{\phi(p)}$$

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CONCLUSION AND FURTHER RESEARCH

Sr. No.	Order of subgroups of G	Number of subgroups of G
1	1	1
2	p	$p^{n-1} + p^{n-2} + p^{n-3} + \cdots + p + 1$
3	p^n	1

By Sylow theorem, there exists a subgroup of order p^2, p^3, …, p^{n-1} of G. So the people can search to find total number of subgroups of order p^2 in G and p^3 in G and so on.

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Source of support: Nil; **Conflict of interest:** Nil.

Cite this article as:

Gawade, K.K. "Counting number of subgroups of order p of = Zp⊕ Zp⊕ …⊕ Zp." *Sarcouncil Journal of Applied Sciences* 1.1 (2021): pp 15-16

Publisher: SARC Publisher