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# Quantum Algorithms for Combinatorial Optimization and Data Analytics: Evaluation of Constraints, Feasibility and Research Trajectories

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**Abstract:** Combinatorial optimization and large-scale data analytics present NP-hard computational challenges, where quantum algorithms promise potential exponential speedups. However, the practical realization of such advantage on Noisy Intermediate-Scale Quantum (NISQ) hardware remains uncertain. This systematic review synthesizes relevant literature to critically evaluate the empirical feasibility and scaling limitations of leading quantum algorithms. The analysis reveals that Variational Quantum Algorithms (VQAs) face persistent reliability challenges such as inconsistent optimality gaps and systemic failures to produce feasible solutions for constrained problems. The principal barrier to VQA scalability is the Measurement Imprecision Wall, where stochastic noise accumulation drives the required measurement shots to scale exponentially with system size, creating a resource deadlock that undermines quantum advantage for large problem instances. For data-centric algorithms, the exponential speedup of Quantum Linear Systems Problem (QLSP) solvers, such as HHL, is largely theoretical for arbitrary classical data due to the Encoding Bottleneck, which requires O(N) runtime for input state preparation. In contrast, Quantum Kernel Methods (QKM) exhibit superior empirical performance and efficient resource scaling, representing the most promising near-term pathway. Achieving practical quantum advantage therefore depends on noise-aware algorithmic design and effective solutions to the classical data ingestion challenge.

**Keywords:** Quantum algorithms, variational quantum algorithms, VQA, quantum kernel methods, HHL algorithm.

## INTRODUCTION

Combinatorial optimization problems such as scheduling for routing and integer programming pervade science and industry. These problems often map to NP-hard decision tasks that are computationally expensive for classical solvers on a large scale (Peres, F., & M. Castelli.). Relatedly, data analytics tasks like clustering and dimensionality reduction underpin modern machine- learning pipelines and increasingly demand faster computational subroutines (like those with sublinear complexity) as dataset sizes grow (Mahdi, *et al.*, 2021).

Quantum computing offers fundamentally different algorithmic primitives that could change how we approach both combinatorial optimization and large-scale data analytics. Hybrid variational frameworks and the Quantum Approximate Optimization Algorithm (QAOA) are designed specifically for such combinatorial optimization problems (Blekos, *et al.*, 2023). Quantum linear-algebra subroutines such as HHL offer potential speedups for solving certain structured systems (Sambhaje, *et al.*, 2024). Quantum-kernel and quantum-feature methods also promise advantages for data- analytics tasks such as dimensionality reduction (Sanjeev, N. 2022; Hrishikesh, D. 2025), by enabling the required computational resources to grow much slower than classical methods as the data size increases.

The quantum advantages for these domains sound promising but they actually remain unsettled in practical implementations. Variational quantum algorithms (VQAs) including QAOA show encouraging behavior on small instances but face hardware noise and scaling uncertainties when run on Noisy Intermediate-Scale Quantum (NISQ) devices (Hrishikesh, D. 2025; Wang, S. *et al.*, 2021)

Similarly, algorithms that claim exponential improvements (e.g., HHL for linear systems) require restrictive assumptions such as favorable condition numbers and efficient quantum data access that limit their direct applicability to practical datasets (Aaronson, S. 2015).

These unresolved practical limitations also shape how research in the area has evolved into a landscape that is far from unified. Different research groups focus separately on QAOA and other variational algorithms, quantum annealing, or quantum machine-learning methods.

They often use different benchmarks and experimental setups, which makes it hard to compare results or assess practical quantum advantages (Schuld, M. *et al.*, 2022). At the same time, results indicate that quantum algorithms can provide super-polynomial or provable advantages for specially constructed optimization instances, even as broad practical speedups remain an open

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research question (Pirnay, N. *et al.*, 2022).

To clarify the current state of knowledge and provide a roadmap for both researchers and practitioners, we perform a systematic review that synthesizes quantum algorithms applied to combinatorial optimization and data analytics. The goals of this systematic review are to:

- Map the landscape of quantum algorithms used for combinatorial optimization and data analytics
- Synthesize empirical performance claims and benchmarks across platforms to identify where reported improvements are reproducible or platform-dependent.
- Critically evaluate the assumptions behind asymptotic speedup proofs and their practical implications
- Identify open challenges and research gaps and provide direction for future work.

## METHODOLOGY

The methodology outlines the research design, data sources and collection strategies, eligibility screening and ethical considerations. This ensures a comprehensive and transparent synthesis of findings.

### Research Design

This study adopted a systematic review methodology to synthesize existing research on quantum algorithms for combinatorial optimization and data analytics. Following established review protocols, this design enabled a structured mapping of algorithmic approaches and performance claims reported across the literature. This approach provides a consolidated evidence base to guide future investigations and inform both research and applied development efforts.

### Data Sources and Collection Strategies

A structured literature search was conducted across established scholarly databases and publisher platforms, including APS Physical Review journals and related conference proceedings. Search terms were designed to reflect the core themes of the review and included combinations such as *"quantum combinatorial optimization," "QAOA," "variational quantum algorithms," "quantum annealing," "quantum machine learning," and "quantum algorithms for data analytics"*.

### Eligibility Criteria

**Inclusion Criteria**

- Peer-reviewed journal articles, conference proceedings, preprints with empirical data, technical reports, or book chapters published between 2015 and 2025.
- Studies presenting empirical or computational results on quantum algorithms for combinatorial optimization or data analytics, including hardware demonstrations (e.g., QAOA or annealing experiments), simulator-based benchmarks, or performance evaluations conducted on NISQ devices or quantum-annealing platforms.

**Exclusion Criteria**

- Non-English language publications.
- Papers published before 2015.
- Opinion pieces, editorials, or blog posts lacking empirical or simulation data.

Following eligibility screening, a total of 30 studies met the inclusion criteria and were included in the final synthesis.

### Ethical Considerations

This review used only publicly available scientific literature and did not involve human subjects or proprietary datasets. Ethical review was therefore not required, following standard practices for systematic reviews in quantum-computing research.

## RESULTS AND FINDINGS

This section presents the empirical performance and practical feasibility of quantum algorithms for combinatorial optimization and data analytics. The analysis emphasizes Variational Quantum Algorithms (VQAs), including the Variational Quantum Eigensolver (VQE) and Quantum Approximate Optimization Algorithm (QAOA), as well as quantum linear-algebra-based methods and quantum kernel approaches. Key constraints, resource requirements, and scalability limitations are evaluated across benchmark studies.

### Empirical Performance in Combinatorial Optimization: Feasibility and Optimality Gaps in Quantum Variants

The systematic assessment of variational quantum algorithms (VQAs), hybrid methods that rely on a classical optimizer to tune the parameters of a quantum circuit, applied to combinatorial optimization reveals considerable theoretical promise (Cerezo de la Roca, *et al.*, 2021).

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However, this promise is accompanied by substantial challenges in implementation feasibility and reliable solution quality. At a structural level, these algorithms operate by encoding a problem Hamiltonian as a weighted sum of elementary operators and iteratively minimizing an observable cost function through repeated quantum measurements and classical feedback, as exemplified by the Variational Quantum Eigensolver (VQE) workflow shown in Figure 1. In practice, this hybrid loop is highly sensitive to hardware noise and circuit depth, leading to deviations between estimated and exact objective values even for small benchmark instances. Hybrid approaches, specifically the VQE and the Quantum Approximate Optimization Algorithm (QAOA), have therefore been systematically evaluated against NP-hard problems commonly encountered in domains such as logistics and finance. These studies demonstrate that training such variational algorithms is itself NP-hard (Lennart, B. & Kliesch, M. 2021; Younis, M. M., A. 2024), motivating a closer examination of their empirical performance and scalability limitations, which are discussed in detail below.

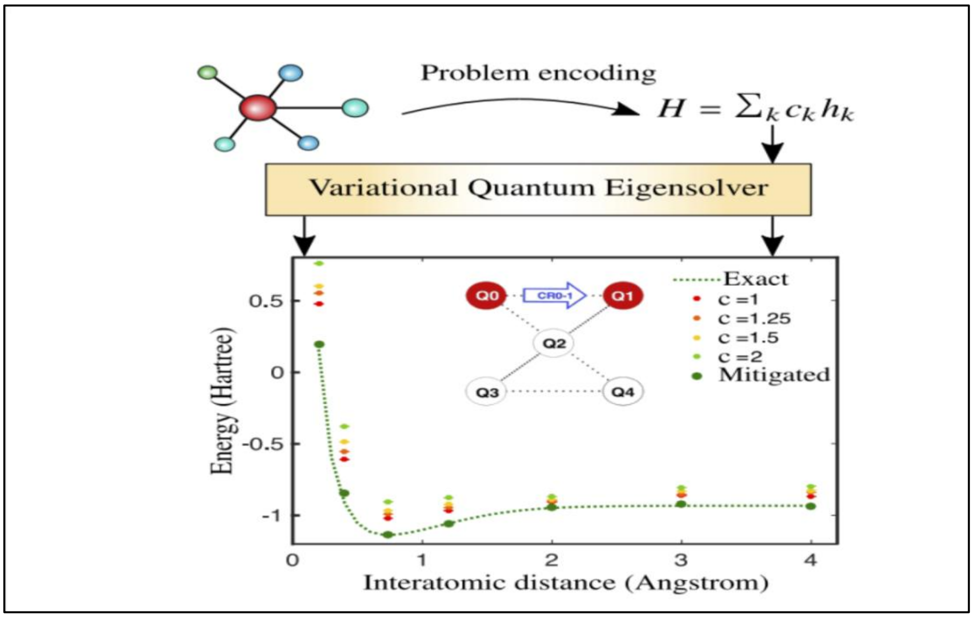


**Figure 1:** Schematic illustration of the Variational Quantum Eigensolver (VQE) workflow for ground-state energy estimation. The problem Hamiltonian is encoded as a weighted sum of operators and optimized via a hybrid quantum–classical loop, with experimental results shown for the $H_2$ molecule on superconducting qubit hardware. Reproduced from Benamer, "Variational Quantum Algorithms: From Theory to NISQ-Era Applications Challenges and Opportunities". Preprints (2025), published under Creative Commons Attribution 4.0 International (CC BY 4.0)

**Benchmarking Variational Quantum Eigensolvers (VQE) and Variants**

Systematic evaluations have benchmarked VQE and its advanced modification, the Conditional Value at Risk-enhanced VQE (CVaR VQE), against standard combinatorial tasks, including the Multi-Dimensional Knapsack Problem (MDKP) and the Maximum Independent Set (MIS) (Lennart, B. & Kliesch, M. 2021). The resulting performance metrics show substantial variability in achieving high-quality solutions. When analyzing the optimality gap (a metric that quantifies the difference between the solution found by the VQA and the true optimal solution for a given problem) for MDKP instances, the results were highly inconsistent compared to known optimal solutions (Lennart, B. & Kliesch, M. 2021). For example, CVaR VQE showed improved outcomes for some instances, significantly reducing the optimality gap. However, this is not a reliable trend; for instance pb5 (problem benchmark 5), the standard VQE achieved a very low optimality gap of 4.25%, which substantially deteriorated to 33.34% when the CVaR enhancement was applied (Lennart, B. & Kliesch, M. 2021).

This variability suggests that current VQAs lack the necessary resilience for generalized optimization deployment. The inconsistent results highlight that small structural changes in

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optimization instances significantly impact the convergence and efficacy of the hybrid quantum-classical loop. The performance heterogeneity indicates that current variational formulations of even enhanced ones like CVaR VQE fail to provide a universally reliable methodology for consistently sampling high-quality solutions across diverse NP-hard landscapes. A more critical limitation arises in constrained optimization problems, where VQE and CVaR VQE demonstrated a systemic failure to respect hard constraints.

Furthermore, in the Market Share Problem (MSP), VQE and CVaR VQE displayed significant constraint violations. This inability to find feasible solutions for inherently constrained problems implies that the common quantum approach of encoding constraints via large penalty terms in the Hamiltonian (H-COPT) is fundamentally insufficient. The classical optimization loop appears to be trapped in regions of the Hilbert space that yield low energy but violate constraints, suggesting that the landscape geometry is dominated by infeasible solutions and demanding the creation of new constraint-aware optimization strategies (Lennart, B. & Kliesch, M. 2021).

One such solution for this limitation was proposed in (Le, T. V., & M. Kliesch. 2024) with the constraint-aware variational quantum eigensolver with constraints (VQEC).

**QAOA: Performance and Computational Overhead**

The Quantum Approximate Optimization Algorithm (QAOA) is also constrained by complex tradeoffs between its theoretical power and required computational resources (Zhong, Qi Zhang. 2026). QAOA exhibits algorithmic sophistication by leveraging non-adiabatic operations to bypass vanishing spectral gaps in problems like MaxCut (Zhou, L. *et al.*, 2018). However, QAOA requires 2^O(p) optimization runs for parameter tuning, resulting in prohibitive computational costs for large instances (Zhou, L. *et al.*, 2018). Empirically, its feasibility and relative solution quality often do not consistently surpass VQE variants (Lennart, B. & Kliesch, M. 2021). Practical deployment is therefore contingent upon efficient heuristic parameter initialization strategies capable of reducing the optimization overhead to O(poly(p)).

**Table 1:** Comparison of Empirical Performance of VQAs for Combinatorial Optimizations

| Problem Class | Algorithm | Key Performance Metric | Key Finding compared Classical Baseline |
|---|---|---|---|
| Multi-Dimensional Knapsack Problem (MDKP) | VQE/CVaR VQE | Optimality gap | Highly inconsistent gap performance. CVaR VQE provides unreliable improvements across instances. |
| Quadratic Assignment Problem (QAP) | VQE/CVaR VQE | Feasibility | Systemic failure to achieve feasible solutions. |
| Market Share Problem (MSP) | VQE/CVaR VQE | Constraint Violation | Significant constraint violations, making MSP the hardest problem for VQA feasibility |
| General Optimization (MaxCut) | QAOA | Optimization Strategy | Can exploit non- adiabatic mechanisms to circumvent small spectral gaps. |
| General Optimization | QAOA/Variants | Computational Expense | Impractical for large problem instances due to training overhead. |

**Quantum Algorithms for Data Analytics and Linear Systems**

Quantum algorithms for data analytics aim to deliver substantial computational advantages, both through linear-algebra primitives such as the Quantum Linear Systems Problem (QLSP) and through quantum kernel methods for supervised machine learning. However, the feasibility of these algorithms hinges on overcoming severe practical constraints related to data input and problem structure.

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**Performance and Resource Efficiency of Quantum Kernel Methods (QKM)**

Quantum Kernel Methods have demonstrated immediate empirical success in quantum machine learning (QML). QKM leverage shallow, variational circuits for quantum feature embedding, showing empirical performance parity or superiority over classical kernels (e.g., RBF) on high-dimensional datasets (Bharti, Kishor, *et al.*, 2022; Jiang, Y., & M. Otten. 2025). For example, the QAmp kernel achieved a 30% improvement over the best classical Linear kernel on the SEED-P12S1 dataset (Jiang, Y., & M. Otten. 2025). Resource requirements are modest, with as few as two qubits for small datasets and O(LN) scaling in circuit depth and gate count, making QKM highly NISQ-compatible.

**Performance and Constraints on Quantum Linear Systems Problem**

HHL achieves O(poly(log N)) theoretical runtime, but exponential speedup is contingent on efficient input state preparation, which often scales as O(N) for generic data (Dervovic, D. 2018; He, G. P. 2025).

Additional limitations arise from the condition number κ of the system matrix, which affects runtime and convergence. Post-HHL algorithms using QSVT, VTAA, or adiabatic techniques mitigate these limitations, reducing κ-dependence to linear scaling (Morales, M. E. S. *et al.*, 2024). Nevertheless, practical deployment remains constrained to well-structured data or quantum-generated inputs.

**Table 2:** Analysis of Quantum Algorithms for Data Processing

| Algorithm | Mechanism / Goal | Key Resource Constraint | Complexity Dependence | Practical Viability |
|---|---|---|---|---|
| HHL (QLSP Solver) | Exponential speedup | Input State Preparation (Encoding) | O(log N) runtime, but O(N) for state prep | Limited to highly structured or quantum-generated data |
| Post-HHL (QSVT, VTAA) | Optimized QLSP Solving | Condition number dependence | Linear in κ | Restricted by input encoding and matrix properties |
| Quantum Kernel Methods (QKM) | Feature embedding for QML | Low qubit count, shallow circuits | O(LN) | High practical viability; empirical advantage on complex datasets |

**Algorithmic and Noise-Induced Scalability Barriers**

Scalability represents the single most significant impediment to achieving practical quantum advantage with VQAs on Near-Intermediate Scale Quantum (NISQ) hardware (Abhinav, K., *et al.*, 2017). This challenge is driven by fundamental limitations imposed by quantum hardware uncertainty and measurement precision, extending beyond purely algorithmic limitations.

**Barren Plateaus and Stochastic Noise**

Training variational quantum algorithms (VQAs) is fundamentally challenged by the Barren Plateau (BP) phenomenon, wherein the gradient of the cost function vanishes exponentially with increasing system size (Giovannini, E.; Morales, M. E. S. *et al.*, 2024). Beyond this intrinsic algorithmic limitation, recent studies demonstrate that stochastic hardware noise introduces an even more severe scaling constraint. As practical problem instances in chemistry and combinatorial optimization require deeper variational ansätze, typically with circuit depth L scaling linearly with the number of qubits n, the presence of local noise causes gradients to decay exponentially in L and, consequently, exponentially in n (Adelina, B. *et al.*, 2025). This effect is visually captured by the progressive flattening of the cost-function landscapes shown in Figure 2, where increasing ansatz depth transforms a well-defined optimization basin into an almost featureless landscape. As a result, even ansätze designed to be theoretically barren-plateau-free exhibit rapidly diminishing optimization signals under realistic noise models. The associated critical noise threshold (σ*) decreases sharply with problem size, indicating that stochastic noise, rather than purely algorithmic design, ultimately dictates the practical trainability limits of VQAs on NISQ hardware (Adelina, B. *et al.*, 2025).

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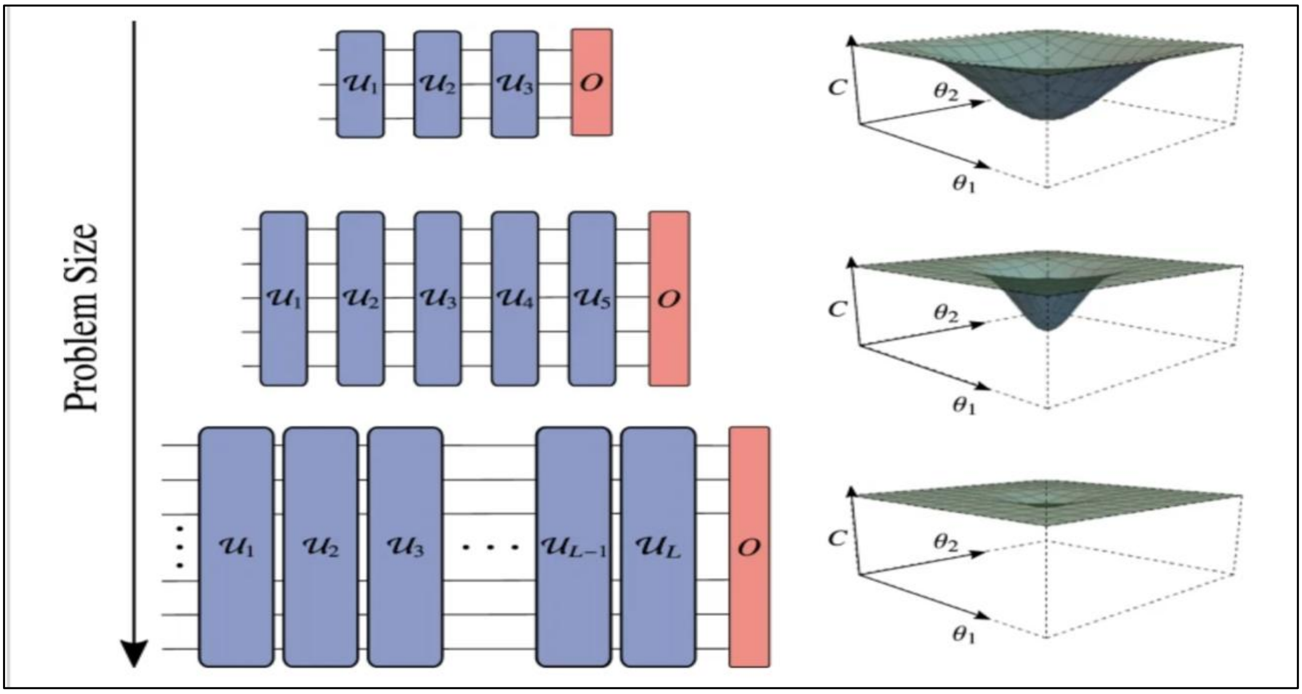


**Figure 2:** Schematic diagram of the Noise-Induced Barren Plateau (NIBP) phenomenon. "Noise-Induced Barren Plateaus in Variational Quantum Algorithms". Reproduced from S. Wang et al. Nature Communications 12, 6961 (2021). Published under CC BY 4.0.

**Measurement Shot Bottleneck**

Achieving sufficient precision in expectation value estimation requires a number of measurement shots that scales prohibitively with system size (Adelina, B. *et al.*, 2025). Consequently, resource requirements may exceed those of classical brute-force sampling for moderately large instances.

**Algorithmic Acceleration**

Advanced strategies, such as counterdiabatic driving, reduce circuit depth and accelerate quantum evolution, improving acceptance rates and solution-space exploration (Dalal, A. *et al.*, 2024; Eder, Peter J. *et al.*, 2025). While promising, these methods are still experimental and limited by NISQ hardware constraints.

**Table 3:** Scalability Constraints in Variational Quantum Algorithms

| **Constraint Domain** | **Mechanism/Phenomenon** | **Impact on Scaling** | **Key Finding** |
|---|---|---|---|
| Algorithmic Training | Barren Plateaus (BP) | Exponential vanishing of gradients with system size/depth | Renders parameter optimization intractable. |
| Hardware Uncertainty | Stochastic Noise (Effective Gaussian) | Critical noise threshold σ decreases rapidly/exponentially with system size $N$ | Confirms deeper, fundamental limitations beyond BP that reduce optimization success probability. |
| Measurement Precision | Required Shot Count | Prohibits polynomial scaling in time/resources | Scales prohibitively bad with $N$, potentially exceeding classical brute- force resources for large problems. |
| Algorithmic Acceleration | Counterdiabatic Driving | Reduces circuit depth and accelerates evolution. | Acceptance rates increase with circuit depth, enabling more efficient exploration |

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## DISCUSSION AND RECOMMENDATIONS

This section synthesizes the empirical findings from variational quantum algorithms (VQAs) and quantum linear systems problem (QLSP) solvers to identify the fundamental obstacles currently preventing the realization of reliable, large-scale quantum advantage on Noisy Intermediate-Scale Quantum (NISQ) hardware. Across both optimization and data-analytics domains, the results consolidate into three critical and interrelated barriers: measurement imprecision driven by stochastic noise, classical data-ingestion bottlenecks, and limited algorithmic resilience in constrained optimization

### The Measurement Imprecision Wall

The most immediate and restrictive barrier to the practical utility of VQAs is the requirement for rapidly increasing measurement resources to counteract stochastic noise on NISQ devices (Adelina, B. *et al.*, 2025; Ito, K. 2023).

**Mechanism:** Empirical and theoretical analyses show that the critical noise threshold ($\sigma^*$) required for successful optimization decreases sharply, potentially exponentially, as the system size N increases (Adelina, B. *et al.*, 2025). Importantly, this behavior persists even for ansatz circuits designed to be theoretically free of barren plateaus, demonstrating that noise-induced limitations are not merely algorithmic but fundamentally hardware-driven (Adelina, B. *et al.*, 2025).

**Impact:** As $\sigma^*$ decreases, the number of measurement shots required to accurately estimate expectation values grows prohibitively with system size. This leads to a resource deadlock, in which the quantum measurement cost approaches or exceeds that of classical brute-force or heuristic methods. Consequently, under current noise characteristics, large-scale quantum advantage for variational optimization appears unattainable without substantial advances in noise mitigation or control (Adelina, B. *et al.*, 2025).

### Classical Data Ingestion and the Encoding Bottleneck

For data-centric quantum algorithms, particularly QLSP solvers such as HHL, the conversion of classical data into quantum states remains a dominant feasibility constraint (He, G. P. 2025).

**Limitation:** While QLSP algorithms promise asymptotic runtimes of O(poly(logN)), this advantage presupposes the efficient preparation of the input state $|b\rangle$. For generic classical data, state preparation typically requires O(N), thereby negating the exponential speedup offered by the quantum solver itself (He, G. P. 2025).

**Consequence:** Until robust and general-purpose state preparation methods with polylogarithmic complexity are demonstrated for classical inputs, QLSP algorithms are effectively constrained to polynomial-time performance. In practice, this limits their applicability to highly structured problems or scenarios where data is natively generated in quantum form, reinforcing the centrality of the input model in determining quantum advantage.

### Algorithmic Resilience in Constrained Optimization

A third critical barrier arises from the limited ability of current VQAs to reliably solve constrained combinatorial optimization problems (Monit, S. & Lau, H. C. 2025).

**Empirical Evidence:** Benchmark results consistently show that VQE and CVaR VQE struggle to produce feasible solutions for heavily constrained problems such as the Quadratic Assignment Problem (QAP) and the Market Share Problem (MSP) (Monit, S. & Lau, H. C. 2025). Even when low- energy solutions are identified, they frequently violate hard constraints.

**Structural Limitation of Penalty Methods:** These failures indicate that the prevailing approach of encoding constraints via large penalty terms in the Hamiltonian (H-COPT) is insufficient. The hybrid optimization loop is often drawn toward low-energy regions dominated by infeasible solutions, suggesting that the underlying optimization landscape is poorly aligned with feasibility requirements (Monit, S. & Lau, H. C. 2025). This limitation highlights the need for new constraint-aware algorithmic paradigms that integrate feasibility directly into the variational search process.

### Recommendations for Future Research Trajectories

Based on the critical barriers identified above, the following research directions are recommended to accelerate progress toward specialized and credible quantum advantage.

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**Prioritize Noise Mitigation and Accelerated Quantum Dynamics**

Given that stochastic noise is the dominant constraint on scalability, research efforts should prioritize techniques that directly reduce noise-induced resource demands. Advanced quantum error mitigation, noise-aware optimization strategies, and control-theoretic approaches that stabilize optimization signals are essential.

In particular, counterdiabatic and accelerated quantum control methods should be further explored. By reducing effective circuit depth and accelerating quantum evolution, these techniques limit the temporal window over which noise accumulates, improving acceptance rates and enhancing exploration of the solution landscape. This approach addresses the measurement imprecision barrier at its physical origin rather than at the level of post- processing.

**Redefine the Input Model for Quantum Data Algorithms**

Progress in quantum data analytics requires a fundamental rethinking of the classical-to-quantum input model. Research should move away from assumptions of generic QRAM availability and instead focus on algorithmic state preparation strategies.

Promising directions include approximate state preparation, structure-exploiting encoding methods, and the recursive use of quantum algorithms themselves to prepare input states more efficiently than O(N). Sustained effort in this direction is necessary before asymptotic speedups for QLSP-style algorithms can translate into practical advantage.

**Capitalize on Quantum Kernel Methods for Near-Term Advantage**

Among all evaluated approaches, Quantum Kernel Methods (QKM) represent the most credible pathway to near-term quantum advantage. Their reliance on shallow, variational circuits, low qubit counts and empirically demonstrated performance gains aligns well with NISQ-era constraints. Future research should focus on systematically characterizing which quantum feature embeddings yield consistent advantages for specific classes of complex data. Optimization of kernel design, robustness analysis, and application-driven benchmarking will be critical for transforming QKM from promising demonstrations into reliable quantum-enhanced tools.

**Concluding Perspective**

Taken together, the results indicate that broad, problem-agnostic quantum advantage remains elusive under current NISQ constraints. However, specialized, resource-aware approaches, particularly those emphasizing noise resilience, input feasibility, and shallow variational structures, offer a realistic and strategically grounded path forward. Rather than pursuing universal speedups, near-term quantum research is best positioned to deliver value through carefully targeted applications where hardware limitations and algorithmic structure are jointly optimized.

## CONCLUSION

This systematic review finds that, despite notable theoretical advances, the practical realization of quantum algorithms for combinatorial optimization and data analytics remains fundamentally constrained by the resource limitations of the Noisy Intermediate-Scale Quantum (NISQ) era.

Empirical evidence shows that current variational quantum algorithms (VQAs) struggle with reliability and scalability. Reported optimality gaps are highly inconsistent across problem instances, and VQAs frequently fail to satisfy hard constraints in constrained optimization tasks. More importantly, scalability is dictated by the accumulation of stochastic noise, which forces the number of required measurement shots to grow prohibitively with system size, imposing strict limits on achievable problem sizes on near-term hardware.

Similarly, quantum linear systems problem (QLSP) solvers promise exponential asymptotic speedups but remain impractical for arbitrary classical data due to the encoding bottleneck. The need to prepare input states with O(N) runtime effectively negates the expected advantage, restricting applicability to highly structured or quantum-generated data.

Despite these limitations, Quantum Kernel Methods (QKM) emerge as a credible near-term pathway. Their reliance on shallow circuits and efficient feature embeddings has enabled empirical performance advantages on high-dimensional datasets within NISQ constraints. Overall, progress is most likely to arise from specialized, noise-aware algorithmic designs and improved input models rather than from broad, problem-agnostic quantum advantage.

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